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RAYSTC: A COMPUTER CODE FOR CALCULATING SINGLE RAY-PATH STATIST--ETC(U)
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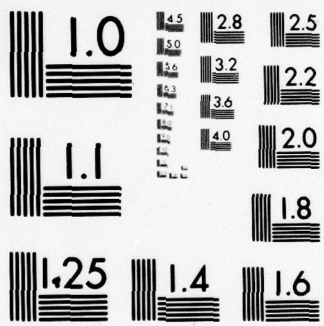

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NRL Report 8304

AD A073762

RAYSTC: A Computer Code for Calculating Single Ray-Path Statistics, Assuming the Garrett-Munk Model of Internal Waves

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July 25, 1979

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Washington, D.C.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Report 8304	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) RAYSTC: A COMPUTER CODE FOR CALCULATING SINGLE RAY-PATH STATISTICS, ASSUMING THE GARRETT-MUNK MODEL OF INTERNAL WAVES		5. TYPE OF REPORT & PERIOD COVERED Interim report on a continuing NRL problem
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Marilyn L. Blodgett		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, D.C. 20375		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62759N, ZF52-552-003, S01-64. 703
11. CONTROLLING OFFICE NAME AND ADDRESS Chief of Naval Material (Direct Funded Programs) Washington, D.C. 20360		12. REPORT DATE July 25, 1979
		13. NUMBER OF PAGES 31
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Acoustic fluctuations	Ocean acoustics
Acoustic propagation	Sound propagation
Computer program	Underwater sound
Internal waves	Wave propagation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes a computer program designed to calculate root-mean-square values for the acoustic phase, phase rate, and log-intensity fluctuation for a single ray path in an ocean in which the sound-speed fluctuations are the result of internal-wave activity. It is assumed that the internal waves can be described by the Garrett-Munk internal-wave model and that the theory of sound propagation developed by Munk and Zachariasen is valid.

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S/N 0102-014-6601

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CONTENTS

RAYSTC: A COMPUTER CODE FOR CALCULATING SINGLE RAY-PATH STATISTICS, ASSUMING THE GARRETT-MUNK MODEL OF INTERNAL WAVES

I. INTRODUCTION

In a recent paper [1] Munk and Zachariasen used the Rytov approximation to calculate the expressions for the mean-square acoustic phase, phase rate, log-intensity, and their spectra for a single ray in an ocean possessing internal-wave induced sound-speed fluctuations. They expressed their results as integrals over the ray path of quantities associated with the internal-wave spectrum. The purpose of this report is to document a computer program designed to evaluate these integrals for the case in which the internal-wave spectrum is given by the Garrett-Munk internal-wave model [2-4].

In Sec. II we briefly outline the theory developed by Munk and Zachariasen and describe the algorithm used in the computer program. Section III contains a description of the program. In Appendix A we indicate the deck assembly and list in a table all the input parameters. In Appendix B the source language listing is given. Appendix C contains the results of a sample run in which we have assumed that the ray path is quadratic in the range variable. This choice of ray path was dictated solely by convenience, and is not necessarily intended to be representative of an actual physical situation.

II. SOME BASIC RELATIONS

According to the general Garrett and Munk internal-wave model [2-4], the vertical particle displacement ζ at depth z has the spectrum

$$F_\zeta(\omega,j;z) = <\zeta^2(z)> G(\omega,\ n(z))\ H(j), \tag{1a}$$

where

$$G(\omega,n) = 0 \text{ for } \omega<\omega_i \text{ and } \omega > n, \tag{1b}$$

$$\int_{\omega_i}^{n} d\omega\ G(\omega,n) = 1, \tag{1c}$$

and

$$\sum_{j=1}^{\infty} H(j) = 1. \tag{1d}$$

Here ω and j are, respectively, the internal-wave radial frequency and the mode number, and ω_i is the inertial frequency. The local buoyancy (Brunt-Väisälä) frequency has the form

$$n(z) = n_o \exp(-z/B) \tag{2}$$

in an exponentially stratified ocean. The mean-square displacement is dependent on depth through $n(z)$, i.e.,

Manuscript submitted December 2, 1979.

$$<\zeta^2(z)> = <\zeta_o^2>\left[\frac{n(z)}{n_o}\right]^{-1}, \tag{3}$$

where $<\zeta_o^2>$ is the mean-square displacement extrapolated to the surface. The horizontal and vertical wave numbers are given by the dispersion relations

$$k_H = \frac{j\pi}{Bn_o}(\omega^2 - \omega_i^2)^{1/2}, \tag{4a}$$

and

$$k_V = \frac{j\pi}{Bn_o}\, n(z) \; . \tag{4b}$$

The analysis in Ref. 1 is based on a version of the Garrett-Munk model [4] GM75 1/2, where

$$G(\omega,n) = N_G\, \omega_i \frac{(\omega^2 - \omega_i^2)^{1/2}}{\omega^3} \; ; \; \omega_i < \omega < n$$

$$= 0 \; ; \text{ otherwise}, \tag{5a}$$

and

$$H(j) = N_H \frac{1}{j^2 + j_*^2} \; . \tag{5b}$$

The mode number parameter j_* is typically set equal to 3. The quantities N_G, N_H are dimensionless normalization constants determined from Eqs. (1c) and (1d):

$$N_G = \frac{4}{\pi}\left[1 + O\left[\frac{\omega_i}{n}\right]\right] \simeq \frac{4}{\pi}, \tag{6a}$$

$$N_H = \left[\sum_{j=1}^{\infty} \frac{1}{j^2 + j_*^2}\right]^{-1} \simeq \frac{2j_*^2}{\pi j_* - 1}. \tag{6b}$$

We are interested in the pressure received at a point $\mathbf{x} = (R_{max},\ 0,\ z)$ in the ocean due to a source located at $\mathbf{x}_s = (0,\ 0,\ z_s)$ and radiating acoustic energy at frequency f. Propagation is, therefore, along the x-axis and R_{max} is the range. (Since we consider the manner in which the rms values accumulate as one moves along the horizontal path from source to receiver, we have appended the subscript *max* to indicate R_{max} is the maximum range of interest.) The acoustic wavenumber is $k_o = 2\pi f / c_o$ where, typically, $c_o = 1.5$ km-Hz.

In the absence of internal waves, the sound speed is equal to the (depth-dependent) mean sound speed:

$$c(\mathbf{x},\ t) = \bar{c}(z) \; , \tag{7}$$

and the received pressure is

$$Re\,[p_o(\mathbf{x})e^{-2\pi i f t}] \; . \tag{8}$$

In Ref. 1 it was assumed p_o could be approximated using ray acoustics. The ray path of interest, $z(x)^{\dagger}$, satisfies the eikonal equation

$$\frac{d^2 z(x)}{dx^2} + V'(z(x)) = 0, \tag{9a}$$

where

$$V'(z) = -\frac{1}{2}\frac{d}{dx}\left[\frac{c_o}{\bar{c}(z)}\right]^2, \tag{9b}$$

and the end-point conditions

$$z(0) = z_s\,,\quad z(R_{\max}) = z. \tag{9c}$$

The slope of the path is given by

$$\tan\theta(x) = \frac{dz(x)}{dx}. \tag{10}$$

(The coordinate system is orientated so that the positive z-direction is downward, hence the ray angle $\theta(x)$ is positive if the ray is directed toward the bottom.)

Also of interest is the phase curvature function [5] $A(x)$ defined by the expression

$$[A(x)]^{-1} = \frac{\xi_1(x)\,\xi_2(x)}{\xi_2(x)\frac{d}{dx}\xi_1(x) - \xi_1(x)\frac{d}{dx}\xi_2(x)}, \tag{11}$$

where $\xi_{1,2}$ are the linearly independent solutions of the differential equation

$$\left\{\frac{d^2}{dx^2} + V''(z(x))\right\}\xi_{1,2} = 0, \tag{12a}$$

with

$$V''(z) = -\frac{1}{2}\frac{d^2}{dz^2}\left[\frac{c_o}{\bar{c}(z)}\right]^2, \tag{12b}$$

that satisfy the boundary conditions

$$\xi_1(0) = \xi_2(R_{\max}) = 0,$$

$$\xi_1(R_{\max}) = \xi_2(0) = 1. \tag{12c}$$

(Because of the Wronskian relation obeyed by $\xi_{1,2}$, the denominator in Eq. (11) is actually independent of x.)

When internal waves are present, the sound speed acquires a small fluctuation $\delta c(\mathbf{x}, t)$ which is related to the internal-wave vertical displacement by

†We have assumed the parabolic approximation. Consequently, the $\sec^2\theta$ term which appears in the expressions for the mean-square values in Ref. 1 has been replaced by unity and the eikonal equation which defines the ray path has a slightly modified form.

$$\frac{\delta c(\mathbf{x},\, t)}{c_o} = (\text{constant})\ n^2(z)\zeta(\mathbf{x},\, t). \tag{13}$$

Therefore, from Eq. (3),

$$\left\langle \left[\frac{\delta c(\mathbf{x},\ t)}{c_o}\right]^2 \right\rangle \equiv \left\langle \left[\frac{\delta c}{c}(z)\right]^2 \right\rangle = \left\langle \left[\frac{\delta c}{c}\right]_o^2 \right\rangle \left[\frac{n(z)}{n_o}\right]^3 . \tag{14}$$

Here, $\left\langle \left[\frac{\delta c}{c}\right]_o^2 \right\rangle$ is the mean-square fractional fluctuation extrapolated to the surface. With the presence of δc, the received pressure can be expressed in the form

$$Re \left[p(\mathbf{x},t)\, e^{-2\pi i f t}\right], \tag{15}$$

where

$$p(\mathbf{x},t) = p_o(\mathbf{x}) \exp\left[X_1(\mathbf{x},t) + i\, X_2(\mathbf{x},t)\right] \tag{16}$$

with $X_{1,2}$ real. This expression can be rewritten as

$$p(\mathbf{x},t) = p_o(\mathbf{x}) \exp\left\{\frac{1}{2}[\iota - <\iota>] + 2\pi i[\phi - <\phi>]\right\}, \tag{17}$$

where anticipating the use of the Rytov approximation we have introduced the definitions

$$\iota \equiv ln|p(\mathbf{x},t)|^2,\ \ \phi \equiv arg\ p(\mathbf{x},t)/2\pi\ ;$$

$$<\iota> \equiv ln|p_o(\mathbf{x},\ t)|^2,\ \ <\phi> \equiv arg\ p_o(\mathbf{x},t)/2\pi\ . \tag{18}$$

(The 2π is included here because we have chosen to measure phase and phase-rate statistics in cycles and cycles-per-hour, respectively.) Mean-square values are given by the expressions

$$\phi_{\text{rms}}^2 = <[\phi - <\phi>]^2>, \tag{19a}$$

$$\dot{\phi}_{\text{rms}}^2 = <\dot{\phi}^2>, \tag{19b}$$

and

$$I_{\text{rms}}^2 = \left[\frac{10}{ln 10}\right]^2 <[\iota - <\iota>]^2> . \tag{19c}$$

Here a dot indicates differentiation with respect to time and I_{rms} is the rms intensity in decibels.

Using the Rytov approximation, Munk and Zachariasen [1] calculated the contribution to these mean-square values from a single ray $z(x)$. They found[†]

$$\phi_{\rm rms}^2 = k_o^2 \int_0^{R_{\max}} dx \left\langle \left| \frac{\delta c}{c} (z(x)) \right|^2 \right\rangle L_p\,(\theta(x),\ z(x))\ , \tag{20a}$$

$$\dot{\phi}_{\rm rms}^2 = k_o^2 \int_0^{R_{\max}} dx \left\langle \left| \frac{\delta c}{c} (z(x)) \right|^2 \right\rangle V_p(\theta(x),\ z(x))\ , \tag{20b}$$

$$I_{\rm rms}^2 = \left|2\pi \frac{10}{ln\,10}\right|^2 k_o^2 \int_0^{R_{\max}} dx \left\langle \left| \frac{\delta c}{c} (z(x)) \right|^2 \right\rangle L_p\,(\theta(x),\ z(x))\ \frac{|A^{-1}(x)|}{k_o L_V^2(z(x))}\ . \tag{20c}$$

In these expressions we have introduced

$$L_p(\theta,\ z) = L(0) f_1 \left| \frac{n(z)}{\omega_i} \tan\theta(z) \right|\ , \tag{21a}$$

$$L(0) \equiv \frac{1}{\pi^4} <j^{-1}> \left|\frac{n_o}{\omega_i}\right| B, \tag{21b}$$

$$f_1(\Delta) \equiv \frac{1}{\Delta^2+1} + \frac{\Delta^2}{2(\Delta^2+1)^{3/2}}\ ln \left\{ \frac{(\Delta^2+1)^{1/2}+1}{(\Delta^2+1)^{1/2}-1} \right\}, \tag{21c}$$

with[‡]

$$V_p(\theta,z) = \frac{8}{\pi^2} <j^{-1}> \omega_i n_o B\ ln \left|\frac{n(z)}{\omega_i}\right| f_2 \left|\frac{n(z)}{\omega_i} \tan\theta\right|, \tag{22a}$$

$$ln\left|\frac{n}{\omega_i}\right| f_2(\Delta) \equiv ln\left|\frac{n}{\omega_i}\right| - \frac{1}{2} ln \left|\frac{\Delta^2}{4}\right| - \frac{1}{2}\ \frac{1}{(\Delta^2+1)^{1/2}}\ ln \left\{ \frac{(\Delta^2+1)^{1/2}+1}{(\Delta^2+1)^{1/2}-1} \right\} \tag{22b}$$

and

$$L_V(z) = \frac{1}{\pi j_*} \left| \frac{<j^{-1}> (\pi j_* - 1)}{\pi} \right|^{1/2} \frac{n_o}{n(z)} B\ . \tag{23}$$

The quantity $<j^{-1}>$ is the average of the reciprocal of the mode number over the internal-wave spectrum[§]:

$$<j^{-1}> = N_H \sum_{j=1}^{\infty} \frac{1}{j}\ \frac{1}{j^2+j_*^2}\ . \tag{24}$$

[†]See footnote on p. 3.

[‡]Properly, one should write f_2 as a function of both Δ and n/ω_i.

[§]Using a technique which we believe to be accurate to within 1 part in 10^6, we find $<j^{-1}>$ = 0.7308, 0.6240, 0.4890, 0.4001, 0.3404, 0.2978 for j_* = 0, 1, 2, 3, 4, 5, respectively.

All the other quantities in Eqs. (20)-(23) have been previously defined. The function $L_p(\theta,z)$ is the sound-speed correlation length measured at depth z along a line inclined at an angle θ with the horizontal, and $L_V(z)$ is the vertical correlation length at depth z. The functions $f_1(\Delta)$ and $f_2(\Delta)$ are defined in such a way as to be equal to unity when Δ is equal to zero. The program assumes n_o and ω_i are given in cycles-per-hour. With this choice, ϕ_{rms} is measured in cycles and $\dot{\phi}_{rms}$ is measured in cycles-per-hour. In the program the integrals Eqs. (20) are calculated using the trapezoidal rule. The range of propagation is broken up into *NR* slabs of equal width, and the integrands in Eqs. (20) are assumed to be linear within each slab. For all the cases we considered, this simple algorithm gave completely satisfactory results.

III. PROGRAM DESCRIPTION

The program was written for use on the Texas Instruments Advanced Scientific Computer in the machine-specific language TI-ASC FORTRAN. Since it is a complete program rather than a subroutine, its use does not require a calling sequence.

The total length of the program is 0001A600. The system reserves an additional 8K words of central memory for I/O buffers, etc. It requires no temporary storage and does not use common blocks. The program will compile on the NX compiler at the *K* level in 1.60 s. The execution time varies. For the sample computer run recorded in Appendix C, the total central processor time was 5.11 s and the plotter time was 14 min.

The program uses the following external routines: ABS, ALOG, ATAN, EXP, FLOAT, INT, SQRT, TAN, ORIGIN, NXAXIS, NYAXIS, LETTER, NUMBER, PLOTS, PLOT, ENDPLT, R$TOP.

The required input data are listed in Table A1 of Appendix A. They naturally fall within four categories: acoustic parameters, internal-wave parameters, output-option parameters, and ray characteristics. The acoustic, internal-wave, and output-option parameters are entered on three separate cards which together compose an input file embedded in the job input stream by means of a START/STOP statement pair. This file has the standard Fortran access name FT05F001 and hence is read on logical unit number 5. Distances are given in meters, the acoustic frequency is given in Hertz, and the frequencies associated with the internal-wave model are given in cycles-per-hour.

The present version of the program does not calculate the ray path nor the phase curvature function. The depth of the ray path as a function of horizontal path length is input by means of a card file specified by a START/STOP pair and having the access name FT08F001 (logical unit number 8). This file consists of the depth of the ray path at NR+1 equally-spaced range points extending from the source position to R_{max} (RMAX). These values are stored in the array RAY in such a way that RAY(I) (I = 1 , ..., NR+1) is the depth of the ray path (in meters) at a horizontal distance (I-1) RMAX/NR from the source. The maximum value of NR allowed by the program is 7000.

It is not the phase curvature function which is required as input but rather the absolute value of its reciprocal. Just as with the ray path, NR+1 values of this function (in meters) compose a card file specified by a START/STOP pair. The file has the access name FT09F001 (logical unit 9). These values are stored in the array ABRECA so that the absolute value at a horizontal distance (I-1) RMAX/NR is ABRECA(I) where I = 1 , ..., NR+1.

On encountering an error condition in the transfer of input data, the program will write out the status code of the error message and terminate the job.

The output from the program can logically be divided into three categories; *i.* input parameters, derived quantities of secondary importance, and results, *ii.* optional tables of various arrays, and *iii.* optional plots. In the following three paragraphs we will describe these categories. All output is written on the standard printer (logical unit 6). The standard access name FT59F001 (logical unit 59) is assigned to the plotter output.

The program lists the input parameters from the first two cards of the input file FT05F001 (Table A1) together with their Fortran names and units. In addition, the source and receiver depths are listed. These depths are the values of the first and last elements of the ray-path array RAY. The program lists the values calculated for the range increment $\Delta r = R_{max}/NR$ (DELR), the wave number k_o (WV), the value for $L(0)$ (LZERO), calculated from Eq. (21b), and the value for $<j^{-1}>$ (AVE). The program then lists the values obtained for the root-mean-square phase, phase-rate, and intensity fluctuations together with the errors associated with the use of the trapezoidal rule.

The user has the option of listing in tables the values of the elements of six arrays for selected values of the indices. These arrays are:

1. RANGE — gives values in meters for the horizontal path length at NR+1 evenly spaced points along the horizontal path of propagation.

2. RAY — contains values for the depth of the ray path.

3. ANGLE — contains the NR values for the ray's grazing angle, in degrees, calculated using a finite difference approximation to the derivative of the ray path.

4. ABRECA — contains values for the magnitude of the phase curvature function. This array is labelled $|(1/A)|$ in the Appendix C table.

5. PHI — contains values for the calculated rms phase fluctuation along the ray path as a function of horizontal distance from the source.

6. PHIDOT — contains values of the rms phase-rate fluctuation as a function of distance from the source.

The extent to which the tables are constructed is determined by the output-option parameters IARR and NPRNT (see Table A1).

Depending on the values assigned the output-option parameters NPLT and IPLT(I), I = 1,...,7, the program will construct up to seven plots. Examples of these plots are given in Appendix C and Table A1 contains brief descriptions.

IV. ACKNOWLEDGMENTS

I would like to thank D. R. Palmer for pointing out the need for this type of program and for helpful advice during its development and documentation.

V. REFERENCES

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Appendix A
DECK ASSEMBLY

Card Number	Description	JSL Statement Format*
1	JOB	/bJOBbName, Acct. no., User code, OPT = (C, R, D, T)
2	LIMIT	/bLIMITbMIN = 1, BAND = 25
3	PLOT FILE DESCRIPTION	/bFDbFT59F001, FORG=PS, RCFM=U,BKSZ=4000,BAND=1/10/1
4	FORTRAN	/bFTNbIN = SDECK, FTVERS = NX, FTNOPT = (K,U)
5	LINK	/bLNK
6	EXECUTE	/bFXQTbOPT = (I,A)
7	PLOT OUTPUT	/bFOSYSbFT59F001, TYPE = PLOT
8	START SOURCE DECK	/bSTARTbACNM = SDECK ⋮ Fortran source deck ⋮
9	STOP SOURCE DECK	/bSTOP
10	START INPUT PARAMETER	/bSTARTbACNM = FT05F001 ⋮ Input parameter cards ⋮
11	STOP INPUT PARAMETER	/bSTOP
12	START RAY	/bSTARTbACNM = FT08F001 ⋮ ray path data cards ⋮
13	STOP RAY	/bSTOP
14	START PHASE CURVATURE	/bSTARTbACNM = FT09F001 ⋮ phase curvature data cards ⋮
15	STOP PHASE CURVATURE	/bSTOP
16	END OF JOB	/bEOJ

*The lower case letter "b" represents a (necessary) blank.

Table A1 — Input Data For RAYSTC

FILE ACCESS NAME FT05F001

CARD 1	Format (F7.2, F8.3, F8.0, I5, F5.0)
CZERO	Reference sound speed c_0 (m/sec)
FREQ	Acoustic frequency f (Hz)
RMAX	Maximum range R_{max} (m)
NR	Number of range steps
DEPTH	Ocean depth D (m)
CARD 2	**Format (F5.0, F5.2, E9.2, F6.4, I5)**
B	Stratification scale B (m)
BVZERO	Buoyancy frequency, extrapolated to the surface, n_0 (cph)
DELC	Root-mean-square value of the fractional sound speed fluctuation, extrapolated to the surface, $<(\delta c/c)_0^2>^{1/2}$
FREQIN	Inertial frequency ω_i (cph)
JSTAR	Mode number parameter j_*
CARD 3	**Format (I1, I4, 8I1)**
IARR	Table output flag = 0; do not print tables = 1; print tables of the arrays RANGE, RAY, ANGLE, ABRECA, PHI, and PHIDOT
NPRNT	Table output count; the tables will be composed of the first, last and every NPRNTth element of the arrays.
NPLT	Plot output flag = 0; construct no plots = 1; construct one or more of the 7 possible plots
IPLT(I)	Individual plot flag = 0; do not construct plot = 1; construct plot

IPLT(1) — plot of $L_p(\theta,z)$ vs θ for various values of z (Fig. 1 of the sample output of App. C)

IPLT(2) — plot of the ray angle θ vs horizontal path length (Fig. 2 of App. C)

IPLT(3) — plot of the ray path (Fig. 3 of App. C)

IPLT(4) — plot of the absolute value of the reciprocal of the phase curvature function (Fig. 4 of App. C)

IPLT(5) — plot of the rms phase fluctuation vs horizontal path length (Fig. 5 of App. C)

IPLT(6) — plot of the rms phase-rate fluctuation vs horizontal path length (Fig. 6 of App. C).

IPLT(7) — plot of $V_p(\theta,z)$ vs θ for various values of z (Fig. 7 of App. C)

FILE ACCESS NAME FT08F001

This file contains a number of cards, format (10F8.1), listing the depth in meters of the ray path at NR+1 evenly spaced points along the horizontal direction of propagation.

FILE ACCESS NAME FT09F001

This card file is analogous to FT08F001 but contains values in meters of the reciprocal of the phase curvature function.

Appendix B

SOURCE LANGUAGE LISTING

```
          PROGRAM RAYSTC
          DIMENSION RAY(7001),ABRECA(7001),AVEJ(6),SLOPE(7000),ANGLE(7000),
         1PHISQ(2,7001),DOTSQ(2,7001),IOTASQ(2,7001),PHI(7001),PHIDOT(7001),
         2PLTAR(1000),IPLT(7)
          DIMENSION XMAX(7),YMIN(7),YMAX(7),YINC(7),NDIGY(7),BCX1(4),BCX2(4)
         1,BCY1(4),BCY2(4),BCY3(2),BCY4(3),BCY5(3),BCY6(3),BCY7(4)
          DATA AVEJ/0.7308,0.6240,0.4890,0.4001,0.3404,0.2978/
          DATA BCI/2HKM/
          DATA NDIGY/-1,1,-1,3,3,3,-1/
          DATA BCX1/4HANGL,4HE(DE,4HGREE,4HS)  /
          DATA BCX2/4HRANG,4HE(KM,4H)   /
          DATA BCY1/4HL(P),4H(M-C,4HYCLE,4H**2)/
          DATA BCY2/4HANGL,4HE(DE,4HGREE,4HS)  /
          DATA BCY3/4HRAY(,4HM)  /
          DATA BCY4/4HABRE,4HCA(K,4HM)  /
          DATA BCY5/4HPHI(,4HCYCL,4HES) /
          DATA BCY6/4HPHID,4HOT(C,4HHP) /
          DATA BCY7/4HV(P),4H(M-C,4HPH**,4H2)  /
          CALL R8STOP
          INTEGER ONE
          REAL*4 LZERO,LP,LVZERO,IOTASQ,IOTOT,MU
          ZERO=0.0
          WRITE(6,97)
    97    FORMAT(1H1)

C  READ IN ACOUSTIC AND INTERNAL WAVE PARAMETERS

          READ(5,1,ERR=4,STV=ISTAT)CZERO,FREQ,RMAX,NR,DEPTH
     1    FORMAT(F7.2,F8.3,F8.0,I5,F5.0)
          READ(5,2,ERR=4,STV=ISTAT)B,BVZERO,DELC,FREQIN,JSTAR
     2    FORMAT(F5.0,F5.2,E9.2,F6.4,I5)
          READ(5,3,ERR=4,STV=ISTAT)IARR,NPRNT,NPLT,(IPLT(I),I=1,7)
     3    FORMAT(I1,I4,8I1)
          GO TO 6
     4    WRITE(6,7)ISTAT
     7    FORMAT(1H ,5HERROR,I10)
          STOP

C  CALCULATE EXPRESSIONS BASED ON INPUT PARAMETERS

     6    DELR=RMAX/FLOAT(NR)
          PI=3.1415927
          PISTAR=PI*JSTAR
          PI2=6.2831853
          WV=(PI2*FREQ)/CZERO
          NRP1=NR + 1
          GAMMA=-1.0/B
```

```
      BETA=BVZERO/FREQIN
      AVE=AVEJ(JSTAR + 1)
      LZERO=(AVE*BETA*B)/(PI**4)
      ALPHA=(((WV*DELC)    )**2)*LZERO*DELR
      VZERO=(32.0*AVE*B*FREQIN*BVZERO)/(PI2*PI2)
      DELTA=(((WV*DELC)    )**2)*VZERO*DELR
      LVZERO=(B/PISTAR)*SQRT((AVE*(PISTAR-1))/PI)
      NU=(((PI2*10)/ALOG(10.))**2)       /(WV*LVZERO*LVZERO)

C  READ IN RAY PATH AND ABSOLUTE VALUE OF RECIPROCAL OF PHASE CURVATURE FUNCTION

      READ(8,11,ERR=4,STV=ISTAT)(RAY(I),I=1,NRP1)
      READ(9,11,ERR=4,STV=ISTAT)(ABRECA(I),I=1,NRP1)
 11   FORMAT(10F8.1)
      ZSCR=RAY(1)
      ZREC=RAY(NRP1)

C  CALCULATE SLOPE AND LOCAL ANGLE RAY PATH MAKES WITH RESPECT TO THE HORIZONTAL

      DO 50 I=1,NR
      SLOPE(I)=(RAY(I+1)-RAY(I))/DELR
      ANGLE(I)=ATAN(SLOPE(I))
 50   CONTINUE

C  CALCULATE PHISQ DOTSQ AND IOTASQ ARRAYS

      DO 10 J=1,2
      PHISQ(J,1)=0.0
      DOTSQ(J,1)=0.0
      IOTASQ(J,1)=0.0
      DO 10 I=1,NR
      T1=GAMMA*RAY(I+J-1)
      T2=EXP(3.0*T1)
      T3=EXP(T1)
      T5=EXP(2.0*T1)
      X4=BETA*T3
      X=X4*ABS(SLOPE(I))

C  CALCULATE F1 AND F2 FUNCTIONS HAVING CALCULATED X

      XSQ=X*X
      IF(XSQ.NE.ZERO)GO TO 30
      FX=1.0
      F2=1.0
      GO TO 32
 30   X1=XSQ+1.0
      X2=SQRT(X1)
```

```
      X3=X2**3
      X5=XSQ/X3
      FX1=1.0/X1
      FX2=ALØG(X2+1.0)
      FX3=X5*FX2
      FX4=.5*X5*ALØG(XSQ)
      FX=FX1+FX3-FX4
      FX5=ALØG(X4)
72    FX6=.5*ALØG  (XSQ/4)
      FX7=(ALØG(X2+1.0)/X2)-(ALØG(XSQ)/(2*X2))
71    F2=(FX5-FX6-FX7)/FX5

32    S=ALPHA*T2*FX
      U=NU*T5*ABRECA(I+J-1)
      PHISQ(J,I+1)=PHISQ(J,I)+S
      IØTASQ(J,I+1)=IØTASQ(J,I)+(S*U)
      DØTSQ(J,I+1)=DØTSQ(J,I )+(DELTA*T2*ALØG(X4)*F2)
10    CØNTINUE
C  CALCULATE RESULTS
      DØ 35 I=1,NRP1
      D1=0.5*(PHISQ(1,I)+PHISQ(2,I))
      D2=0.5*(DØTSQ(1,I)+DØTSQ(2,I))
      PHI(I)=SQRT(D1)
      PHIDØT(I)=SQRT(D2)
35    CØNTINUE
      PHITØT=PHI(NRP1)
      DØTTØT=PHIDØT(NRP1)
      IØTØT=SQRT(0.5*(IØTASQ(1,NRP1)+ IØTASQ(2,NRP1)))
C  CALCULATE ERRØR
      IF(PHITØT.EQ.ZERØ)GØ TØ 53
      EPHI=100.00*ABS(PHISQ(1,NRP1)-PHISQ(2,NRP1))/(PHITØT*PHITØT)
56    IF(DØTTØT.EQ.ZERØ)GØ TØ 54
      EDØT=100.00*ABS(DØTSQ(1,NRP1)-DØTSQ(2,NRP1))/(DØTTØT*DØTTØT)
57    IF(IØTØT.EQ.ZERØ)GØ TØ 55
      EIØT=100.00*ABS(IØTASQ(1,NRP1)-IØTASQ(2,NRP1))/(IØTØT*IØTØT)
      GØ TØ 52
53    EPHI=0.0
      GØ TØ 56
54    EDØT=0.0
      GØ TØ 57
55    EIØT=0.0
C  PRINT ØUT INPUT PARAMETERS, CALCULATED QUANTITIES AND RESULTS
```

```
0112      52   WRITE(6,300)
0113     300   FORMAT(1H1,100HPROGRAM FOR CALCULATING SINGLE RAY PATH STATISTICS
              1ASSUMING THE GARRETT-MUNK MODEL OF INTERNAL WAVES/)
0114           WRITE(6,301)
0115     301   FORMAT(1H0,25HINPUT ACOUSTIC PARAMETERS/)
0116           WRITE(6,302)CZERO
0117     302   FORMAT(1H ,33HREFERENCE SOUND SPEED (CZERO) =  ,F7.2,6H M/SEC)
0118           WRITE(6,303)FREQ
0119     303   FORMAT(1H ,33HACOUSTIC FREQUENCY (FREQ)     =  ,F8.3,6H HERTZ)
0120           WRITE(6,304)RMAX
0121     304   FORMAT(1H ,33HMAX RANGE (RMAX)              =  ,F8.0,2H M)
0122           WRITE(6,305)NR
0123     305   FORMAT(1H ,33HNUMBER OF RANGE STEPS (NR)    =  ,I5)
0124           WRITE(6,306)DEPTH
0125     306   FORMAT(1H ,33HOCEAN DEPTH (DEPTH)           =  ,F5.0,2H M)
0126           WRITE(6,307)ZSCR
0127     307   FORMAT(1H ,33HSOURCE DEPTH (ZSCR)           =  ,F6.1,2H M)
0128           WRITE(6,308)ZREC
0129     308   FORMAT(1H ,33HRECEIVER DEPTH (ZREC)         =  ,F6.1,2H M)
0130           WRITE(6,309)
0131     309   FORMAT(1H0,30HINPUT INTERNAL WAVE PARAMETERS)
0132           WRITE(6,310)B
0133     310   FORMAT(1H0,42HSTRATIFICATION SCALE (B)                   =  ,F5.0,2H M
              1)
0134           WRITE(6,311)BVZERO
0135     311   FORMAT(1H ,42HEXTRAPOLATED B-V FREQUENCY (BVZERO)    =  ,F5.2,4H C
              1PH)
0136           WRITE(6,312)DELC
0137     312   FORMAT(1H ,41HEXTRAPOLATED FRAC. FLUCTUATION (DELC)  = ,E9.2)
0138           WRITE(6,313)FREQIN
0139     313   FORMAT(1H ,42HINERTIAL FREQUENCY (FREQIN)                =  ,F6.4,4H C
              1PH)
0140           WRITE(6,314)JSTAR
0141     314   FORMAT(1H ,42HJSTAR (JSTAR)                              =  ,I1)
0142           WRITE(6,315)
0143     315   FORMAT(1H0,21HCALCULATED QUANTITIES)
0144           WRITE(6,316)DELR
0145     316   FORMAT(1H0,21HRANGE INCR. (DELR) = ,F7.2,2H M)
0146           WRITE(6,317)WV
0147     317   FORMAT(1H ,21HWAVENUMBER (WV)    = ,F6.4,3H /M)
0148           WRITE(6,318)LZERO
0149     318   FORMAT(1H ,21HLZERO              = ,F8.3,3H  M)
0150           WRITE(6,319)AVE
0151     319   FORMAT(1H ,21HAVE OF 1/J (AVE)   = ,F6.4)
0152           WRITE(6,320)
0153     320   FORMAT(1H0,7HRESULTS)
```

```
      WRITE(6,321)PHITOT
  321 FORMAT(1H0,41HRMS PHASE FLUCTUATION (PHITOT)         = ,F9.5,7H C
     1YCLE)
      WRITE(6,322)DOTTOT
  322 FORMAT(1H ,41HRMS PHASE RATE FLUCTUATION (DOTTOT)    = ,F9.5,5H C
     1PM)
      WRITE(6,323)IOTOT
  323 FORMAT(1H.,41HRMS INTENSITY FLUCTUATION (IOTOT)      = ,F7.3,6H
     1 DB)
      WRITE(6,324)EPHI
  324 FORMAT(1H0,50HERROR FOR PHASE  (EPHI)     =      ,F
     16.2,8H PERCENT)
      WRITE(6,325)EDOT
  325 FORMAT(1H ,50HERROR FOR PHASE RATE (EDOT) =      ,F
     16.2,8H PERCENT)
      WRITE(6,326)EIOT
  326 FORMAT(1H ,50HERROR FOR INTENSITY  (EIOT)  =     ,
     1F6.2,8H PERCENT)
C  CHECK IF ARRAYS TO BE PRINTED OUT
      IF(IARR.EQ.0) GO TO 60
C  WRITE OUT TABLES
      WRITE(6,327)
  327 FORMAT(1H0,6HTABLES)
      WRITE(6,328)
  328 FORMAT(1H0,95HINDEX    RANGE-M    RAY(I)-M    ANGLE(I-1)-RAD    |(
     1I/A)|(I)-M     PHI(I)-CYCLE     PHIDOT(I)-CPM)
      ONE=1.0
      WRITE(6,329)ONE,ZERO,RAY(1),ABRECA(1)    ,PHI(1),PHIDOT(1)
  329 FORMAT(1H0,I5,4X,F8.0,3X,F8.1,24X,F8.1,7X,F9.5,6X,F9.5)
      DO 16 K=NPRNT,NRP1,NPRNT
      SMR=(K-1)*DELR
      WRITE(6,330)K,SMR,RAY(K),ANGLE(K-1),ABRECA(K)    ,PHI(K),PHIDOT(K)
  330 FORMAT(1H ,I5,4X,F8.0,3X,F8.1,4X,F8.5,12X,F8.1,7X,F9.5,6X,F9.5)
   16 CONTINUE
      IF(K.EQ.NRP1)GO TO 60
      SMR=NR*DELR
      WRITE(6,330)NRP1,SMR,RAY(NRP1),ANGLE(NRP1-1),ABRECA(NRP1),PHI(NRP1
     1),PHIDOT(NRP1)
C  CHECK IF PLOTS REQUIRED
   60 IF(NPLT.EQ.0) GO TO  98
      CALL PLOTS(PLTAR,1000,.75)
```

```
      IFIRST=1
C  CALCULATE MINIMUM AND MAXIMUM VALUES ATTAINED BY PLOTS
      DO 999 K=1,7
      YMIN(K)=0.0
      IF(K.EQ.2) YMIN(K)=-20.0
      IF(K.EQ.3) YMIN(K)=DEPTH
  999 CONTINUE
      YMAX(1)=10.0*INT((1.1*LZERO)/10.0) + 10.0
      YMAX(2)=20.0
      YMAX(3)=0.0
      TOP=ABRECA(1)
      DO 997 K=1,NR
      COMP=ABRECA(K+1)
      IF(COMP.GT.TOP) TOP=COMP
  997 CONTINUE
      YMAX(4)=1.1*(TOP/1000.)
      YMAX(5)=1.1*PHITOT
      YMAX(6)=1.1*DOTTOT
      YMAX(7)=10.0*INT(VZERO*(ALOG(BETA)+1)/10.0) + 10.0
      DO 996 K=1,7
      YINC(K)=(YMAX(K)-YMIN(K))/10.0
      IF(K.NE.2) GO TO 996
      YINC(2)=5.0
  996 CONTINUE
      DO 998 K=1,7
      XMAX(K)=RMAX/1000.0
      IF((K.EQ.1).OR.(K.EQ.7))XMAX(K)=10.0
  998 CONTINUE
C  DRAW PLOTS
      DO 700 K=1,7
      IF(IPLT(K).EQ.0) GO TO 700
C  DRAW X AND Y AXES
      BMAX=XMAX(K)
      ZMIN=YMIN(K)
      ZMAX=YMAX(K)
      XIN=BMAX/10.0
      YIN=YINC(K)
      NX=-1
      NY=NDIGY(K)
      YSCALE=(ZMAX-ZMIN)/10.0
      XSCALE=XIN
```

```
      IPEN=3
      XADD=DELR/1000./XSCALE
      IF(IFIRST.NE.1) GO TO 15
      IFIRST=0
      CALL ORIGIN(2.0,.0)
15    CALL NXAXIS(0.0,0.0,XIN,BMAX,10.,-.07,.07,NX)
      IF(K.EQ.1.OR.K.EQ.7) GO TO 13
      CALL LETTER(4.0,-.5,.25,BCX2,0.,12)
13    CALL NYAXIS(0.0,ZMIN,YIN,ZMAX,  10.0 ,-.07,.07,NY)
      CALL NUMBER(-.125,-.125,.07,ZMIN,90.,NY)
      CALL NXAXIS(0.,10.0,0.,XIN,BMAX,10.0,.07,0,NX)
      CALL NYAXIS(10.0,0.,ZMIN,YIN,ZMAX,10.0,.07,0,NY)
      GO TO(801,802,803,804,805,806,807),K

C  PLOT 1 L(P)VRS 0

801   ILBL=1
      CALL LETTER(3.5,-.50,.25,BCX1,0,16)
      CALL LETTER(-.375,3.0,.25,BCY1,90.,16)
      RAD=PI/180.
      XADD=0.2/XSCALE
      DO 63 J=0,2000,250
      IF(J.EQ.1250.OR.J.EQ.1750) GO TO 63
      ILBL=ILBL + 5
      IPEN=3
      XAXIS=-XADD
      T1=EXP(GAMMA*J)*BETA
      DO 25 I=1,51
      THETA=(I-1)*.2*RAD
      X=TAN(THETA) * T1

C  CALCULATE F1 FUNCTION HAVING CALCULATED X

      XSQ=X*X
      IF(XSQ.NE.ZERO)GO TO 40
      FX=1.0
      GO TO 41
40    X1=XSQ + 1.0
      X2=SQRT(X1)
      X3=X2**3
      X5=XSQ/X3
      FX1=1.0/X1
      FX2=ALOG(X2+1.0)
      FX3=X5*FX2
      FX4=.5*X5*ALOG(XSQ)
      FX=FX1+FX3-FX4
41    XAXIS=(XAXIS+XADD)
```

```
      LP=(LZERO*FX)
      IF(LP.LT.0.0)LP=0.0
      YAXIS=LP/YSCALE
      CALL PLOT(XAXIS,YAXIS,IPEN)
      IPEN=2
      IF(ILBL.EQ.I) GO TO 95
      GO TO 25
95    ZNUM=J/1000.
      YAX=YAXIS + .03
      CALL NUMBER(XAXIS + .03,YAX  ,.07,ZNUM,0,2)
      CALL LETTER(XAXIS + .33,YAX  ,.07,BCI,0,2)
      CALL PLOT(XAXIS,YAXIS,3)
25    CONTINUE
63    CONTINUE
      GO TO 61

C  PLOT 2  ANGLE(R) VRS R

802   XAXIS=0
      CALL LETTER(-.375,3.0,.25,BCY2,90.,16)
      DO 21 I=1,NR
      XAXIS=(XAXIS+XADD)
      DEG=180./PI
      DEGREE=ANGLE(I)*DEG
      IF(DEGREE.GT.20.0)GO TO 12
      IF(DEGREE.LT.-20.0)DEGREE=-20.0
      GO TO 23
12    DEGREE=20.0
23    YAXIS=(DEGREE-ZMIN)/YSCALE
      CALL PLOT(XAXIS,YAXIS,IPEN)
      IPEN=2
21    CONTINUE
      GO TO 61

C  PLOT 3  RAY(R) VRS R

803   XAXIS=-XADD
      CALL LETTER(-.375,4.0,.25,BCY3,90.,8)
      DO 20 I=1,NRP1
      XAXIS=(XAXIS+XADD)
      IF(RAY(I).LT.0.0)RAY(I)=0.0
      IF(RAY(I).GT.DEPTH)RAY(I)=DEPTH
      YAXIS=(RAY(I)-ZMIN)/YSCALE
      CALL PLOT(XAXIS,YAXIS,IPEN)
      IPEN=2
20    CONTINUE
      GO TO 61
```

```
C  PLOT 4  ABRECA(R) VRS R

  804  XAXIS=-XADD
       CALL LETTER(-.375,4.0,.25,BCY4,90.,12)
       DO 605 I=1,NRP1
       XAXIS=XAXIS + XADD
       YAXIS=((ABRECA(I)/1000.)-ZMIN)/YSCALE
       CALL PLOT(XAXIS,YAXIS,IPEN)
       IPEN=2
  605  CONTINUE
       GO TO 61

C  PLOT 5  PHI(R) VRS R

  805  XAXIS=-XADD
       CALL LETTER(-.375,4.0,.25,BCY5,90.,12)
       DO 22 I=1,NRP1
       XAXIS=(XAXIS+XADD)
       IF(PHI(I).LT.0.0)PHI(I)=0.0
       YAXIS=(PHI(I)-ZMIN)/YSCALE
       CALL PLOT(XAXIS,YAXIS,IPEN)
       IPEN=2
  22   CONTINUE
       GO TO 61

C  PLOT 6  PHIDOT(R) VRS R

  806  XAXIS=-XADD
       CALL LETTER(-.375,4.0,.25,BCY6,90.,12)
       DO 65 I=1,NRP1
       XAXIS=(XAXIS+XADD)
       IF(PHIDOT(I).LT.0.0)PHIDOT(I)=0.0
       YAXIS=(PHIDOT(I)-ZMIN)/YSCALE
       CALL PLOT(XAXIS,YAXIS,IPEN)
       IPEN=2
  65   CONTINUE
       GO TO 61

C  PLOT 7 V(P) VRS O

  807  RAD=3.14159265/180.
       CALL LETTER(3.5,-.50,.25,BCX1,0,16)
       CALL LETTER(-.375,3.0,.25,BCY7,90.,16)
       XADD=0.2/XSCALE
       DO 83 J=0,2000,250
       IF(J.EQ.1250.OR.J.EQ.1750) GO TO 83
```

```
      IPEN=3
      XAXIS=-XADD
      T1=EXP(GAMMA*J)*BETA
      T2=ALOG  (T1)
      DO 85 I=1,51
      THETA=(I-1)*.2*RAD
      X=TAN(THETA)*T1
C  CALCULATE F2 FUNCTION HAVING CALCULATED X
      XSQ=X*X
      IF(XSQ.NE.ZERO)GO TO 90
      F2=1.0
      GO TO 91
   90 X1=XSQ + 1.0
      X2=SQRT(X1)
      FX5=ALOG(T1)
      FX6=.5*ALOG  (XSQ/4)
      FX7=(ALOG(X2+1.0)/X2)-(ALOG(XSQ)/(2*X2))
      F2=(FX5-FX6-FX7)/FX5
   91 XAXIS=XAXIS + XADD
      VP=VZERO*T2*F2
      IF(VP.LT.0.0)VP=0.0
      YAXIS=VP/YSCALE
      CALL PLOT(XAXIS,YAXIS,IPEN)
      IPEN=2
      IF(I.NE.4) GO TO 85
      ZNUM=J/1000.
      YAX=YAXIS + .06
      CALL NUMBER(XAXIS + .03,YAX,.07,ZNUM,0,2)
      CALL LETTER(XAXIS + .33,YAX,.07,BCI,0,2)
      CALL PLOT(XAXIS,YAXIS,3)
   85 CONTINUE
   83 CONTINUE
   61 CALL ORIGIN(12.0,0)
  700 CONTINUE
      CALL ENDPLT
   98 STOP
      END
```

Appendix C
SAMPLE RUN

```
/ JOB MARILYN BLODGETT,          BLODM1,OPT=(C,R,D,T)
/ LIMIT MIN=1,BAND=25
/ FD FT59F001,FORG=PS,RCFM=U,BKSZ=4000,BAND=1/10/1
/ FTN IN=SDECK,FTVERS=NX,FTNOPT=(K,U)
/ LNK
/ FXQT OPT=(I,A)
/ FOSYS FT59F001,TYPE=PLOT
/ START ACNM=SDECK
.
.
.
FORTRAN SOURCE DECK
.
.
.
/ STOP
/ START ACNM=FT05F001
1500.00 100.000  10000.  1004040.
1000. 3.00 0.49E-030.0420     3
1  1011111111
/ STOP
/ START ACNM=FT08F001
  1000.0   960.4   921.6   883.6   846.4   810.0   774.4   739.6   705.6   672.4
   640.0   608.4   577.6   547.6   518.4   490.0   462.4   435.6   409.6   384.4
   360.0   336.4   313.6   291.6   270.4   250.0   230.4   211.6   193.6   176.4
   160.0   144.4   129.6   115.6   102.4    90.0    78.4    67.6    57.6    48.4
    40.0    32.4    25.6    19.6    14.4    10.0     6.4     3.6     1.6     0.4
     0.0     0.4     1.6     3.6     6.4    10.0    14.4    19.6    25.6    32.4
    40.0    48.4    57.6    67.6    79.4    90.0   102.4   115.6   129.6   144.4
   160.0   176.4   193.6   211.6   230.4   250.0   270.4   291.6   313.6   336.4
   360.0   384.4   409.6   435.6   462.4   490.0   518.4   547.6   577.6   608.4
   640.0   672.4   705.6   739.6   774.4   810.0   846.4   883.6   921.6   960.4
  1000.0
/ STOP
/ START ACNM=FT09F001
     0.0    99.0   196.0   291.0   384.0   475.0   564.0   651.0   736.0   819.0
   900.0   979.0  1056.0  1131.0  1204.0  1275.0  1344.0  1411.0  1476.0  1539.0
  1600.0  1659.0  1716.0  1771.0  1824.0  1875.0  1924.0  1971.0  2016.0  2059.0
  2100.0  2139.0  2176.0  2211.0  2244.0  2275.0  2304.0  2331.0  2356.0  2379.0
  2400.0  2419.0  2436.0  2451.0  2464.0  2475.0  2484.0  2491.0  2496.0  2499.0
  2500.0  2499.0  2496.0  2491.0  2484.0  2475.0  2464.0  2451.0  2436.0  2419.0
  2400.0  2379.0  2356.0  2331.0  2304.0  2275.0  2244.0  2211.0  2176.0  2139.0
  2100.0  2059.0  2016.0  1971.0  1924.0  1875.0  1824.0  1771.0  1716.0  1659.0
  1600.0  1539.0  1476.0  1411.0  1344.0  1275.0  1204.0  1131.0  1056.0   979.0
   900.0   819.0   736.0   651.0   564.0   475.0   384.0   291.0   196.0    99.0
     0.0
/ STOP
/ EOJ
```

PROGRAM FOR CALCULATING SINGLE RAY PATH STATISTICS ASSUMING THE GARRETT-MUNK MODEL OF INTERNAL WAVES

INPUT ACOUSTIC PARAMETERS

```
REFERENCE SOUND SPEED (CZERO) =  1500.00 M/SEC
ACOUSTIC FREQUENCY (FREQ)     =   100.000 HERTZ
MAX RANGE (RMAX)              =    10000. M
NUMBER OF RANGE STEPS (NR)    =    100
OCEAN DEPTH (DEPTH)           =  4040. M
SOURCE DEPTH (ZSCR)           =  1000.0 M
RECEIVER DEPTH (ZREC)         =  1000.0 M
```

INPUT INTERNAL WAVE PARAMETERS

```
STRATIFICATION SCALE (B)                   =  1000. M
EXTRAPOLATED B-V FREQUENCY (BVZERO)        =   3.00 CPH
EXTRAPOLATED FRAC. FLUCTUATION (DELC)      =  0.49E-03
INERTIAL FREQUENCY (FREQIN)                =  0.0420 CPH
JSTAR (JSTAR)                              =  3
```

CALCULATED QUANTITIES

```
RANGE INCR. (DELR) =  100.00 M
WAVENUMBER (WV)    = 0.4189 /M
LZERO              =  293.307  M
AVE OF 1/J (AVE)   = 0.4001
```

RESULTS

```
RMS PHASE FLUCTUATION (PHITOT)          =    0.10065  CYCLE
RMS PHASE RATE FLUCTUATION (DOTTOT)     =    0.16132  CPH
RMS INTENSITY FLUCTUATION (IOTOT)       =    1.006    DB

ERROR FOR PHASE  (EPHI)       =     0.00 PERCENT
ERROR FOR PHASE RATE (EDOT)   =     0.00 PERCENT
ERROR FOR INTENSITY  (EIOT)   =     0.00 PERCENT
```

TABLES

INDEX	RANGE-M	RAY(I)-M	ANGLE(I-1)-RAD	](1/A)](I)-M	PHI(I)-CYCLE	PHIDOT(I)-CPH
1	0.	1000.0		0.0	0.00000	0.00000
10	900.	672.4	-0.32055	819.0	0.00301	0.01512
20	1900.	384.4	-0.24686	1539.0	0.00708	0.03089
30	2900.	176.4	-0.17033	2059.0	0.01179	0.05205
40	3900.	48.4	-0.09174	2379.0	0.02075	0.07875
50	4900.	0.4	-0.01200	2499.0	0.06122	0.11076
60	5900.	32.4	0.06790	2419.0	0.09778	0.13730
70	6900.	144.4	0.14693	2139.0	0.09980	0.15093
80	7900.	336.4	0.22417	1659.0	0.10035	0.15756
90	8900.	608.4	0.29870	979.0	0.10056	0.16033
100	9900.	960.4	0.37012	99.0	0.10065	0.16120
101	10000.	1000.0	0.37705	0.0	0.10065	0.16132

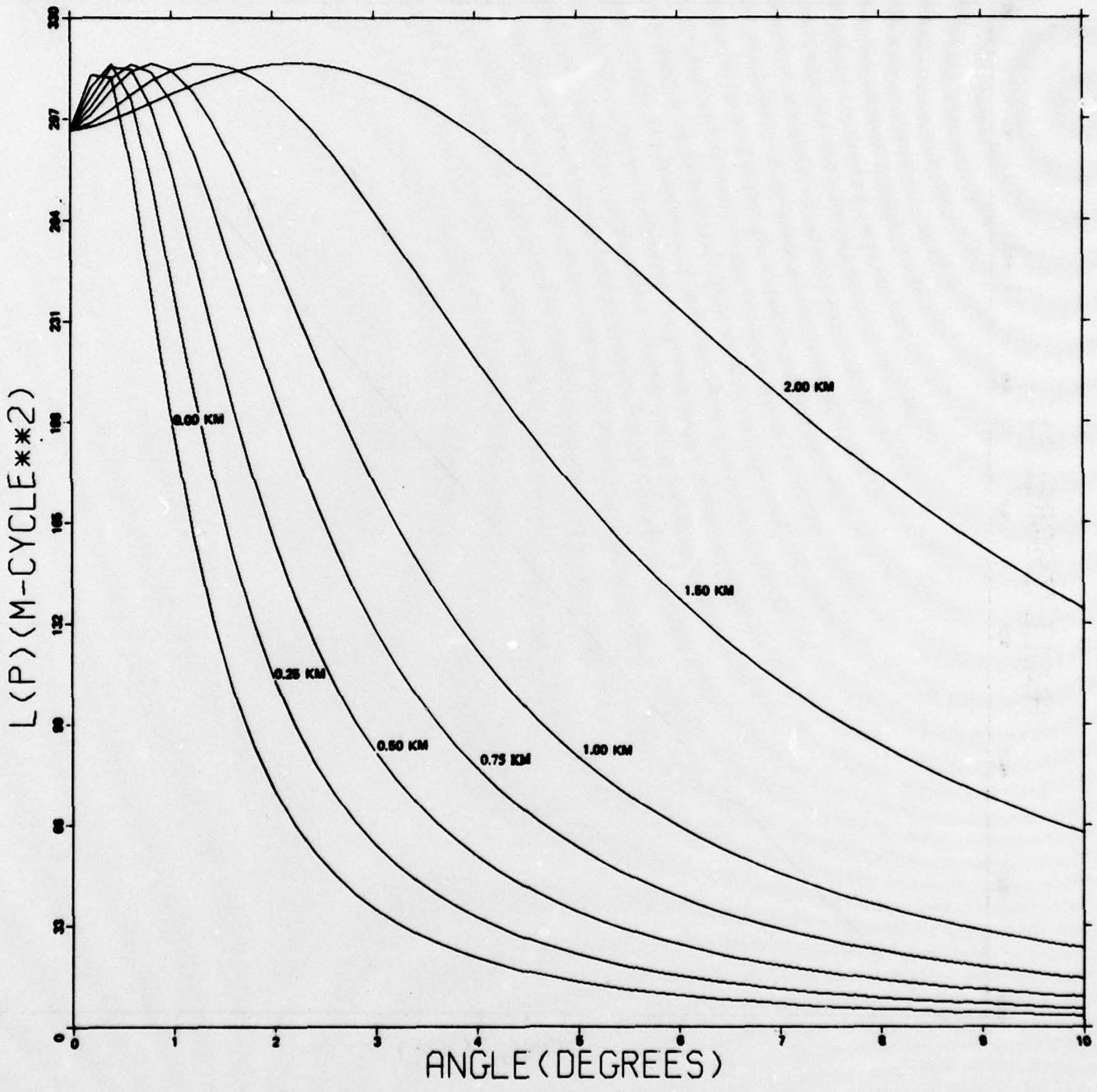

Fig. C1—Plot of $L_p(\theta,z)$ vs θ for various values of z

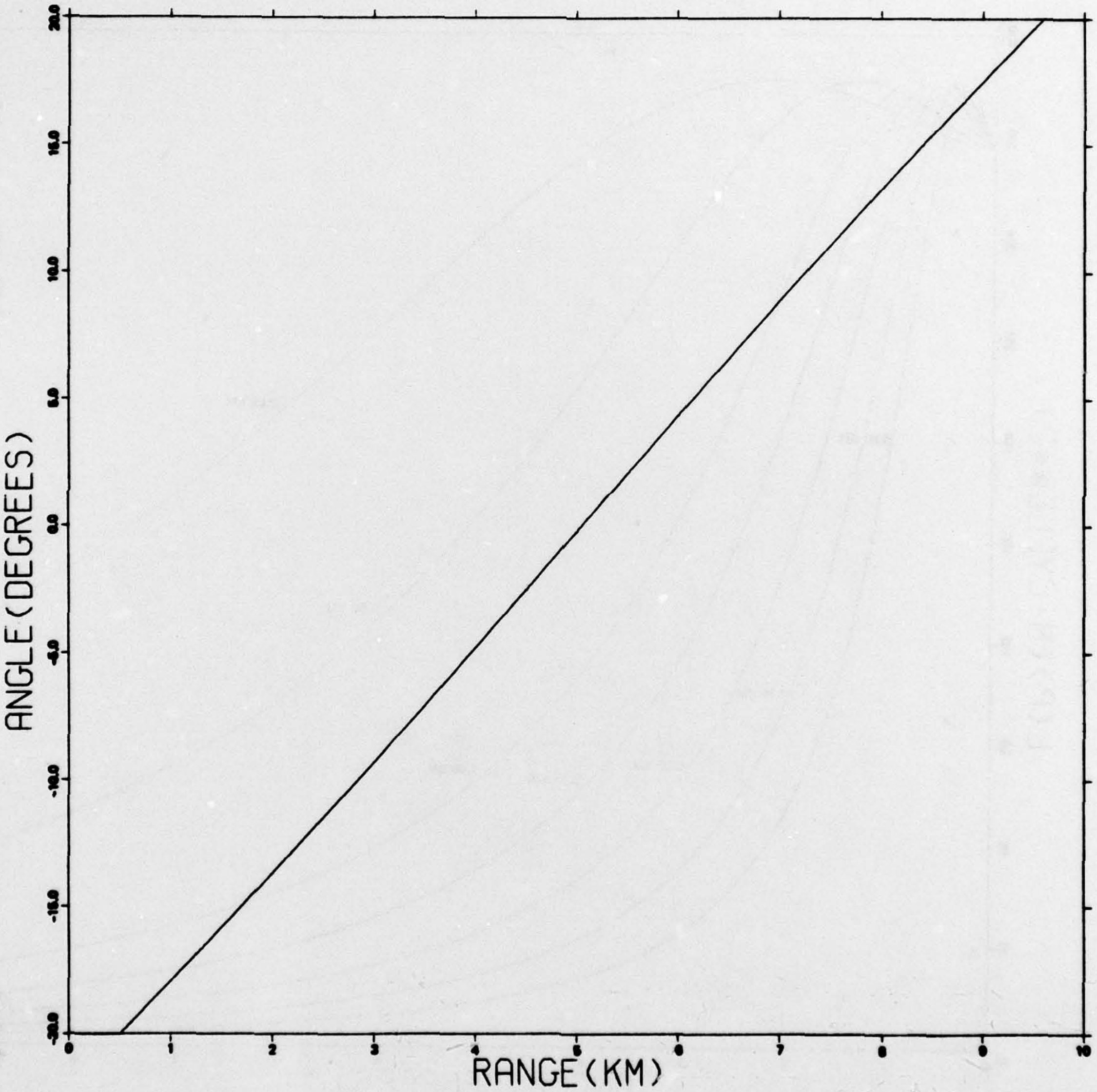

Fig. C2—Plot of the ray angle θ vs horizontal path length

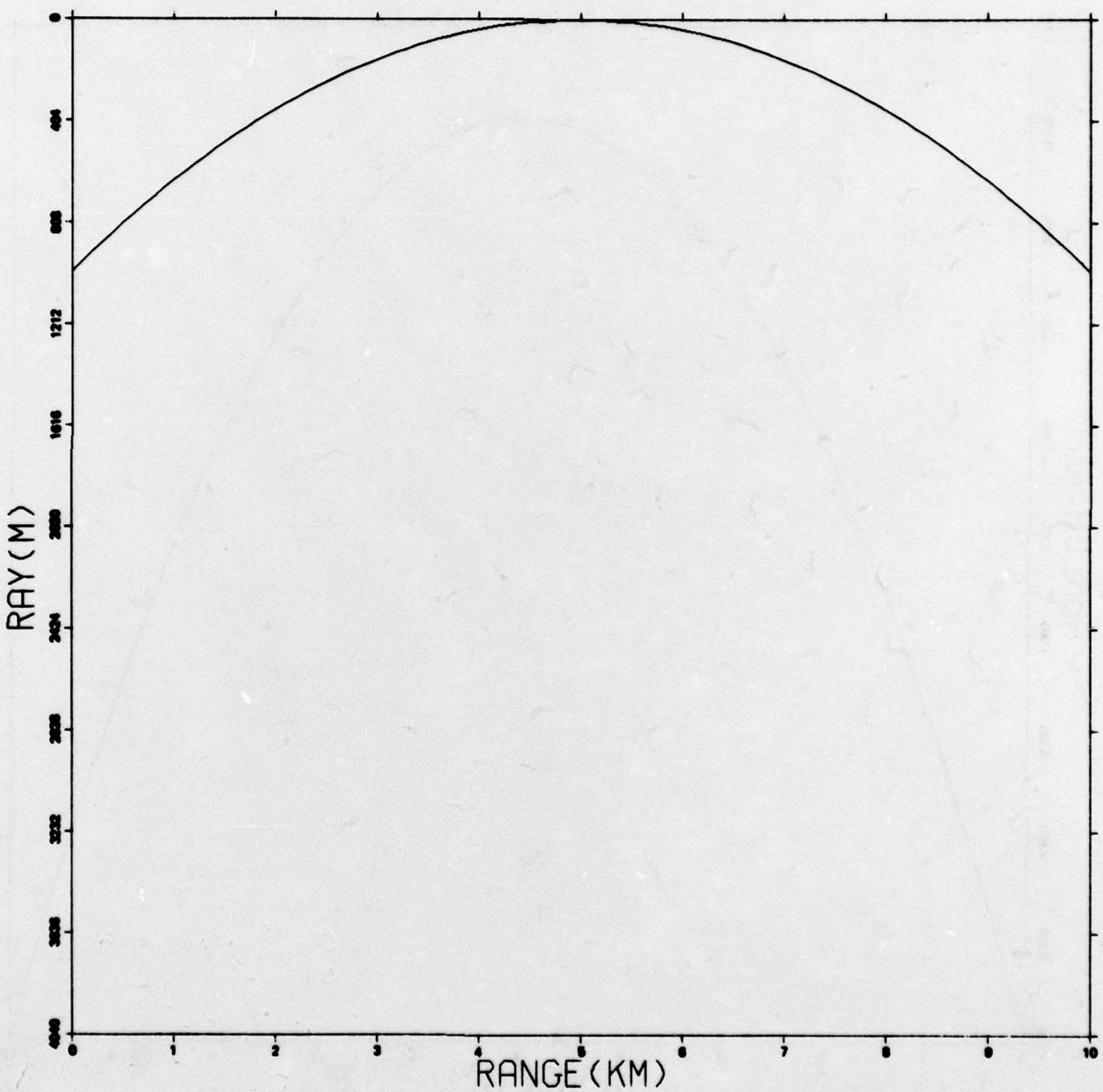

Fig. C3—Plot of the ray path

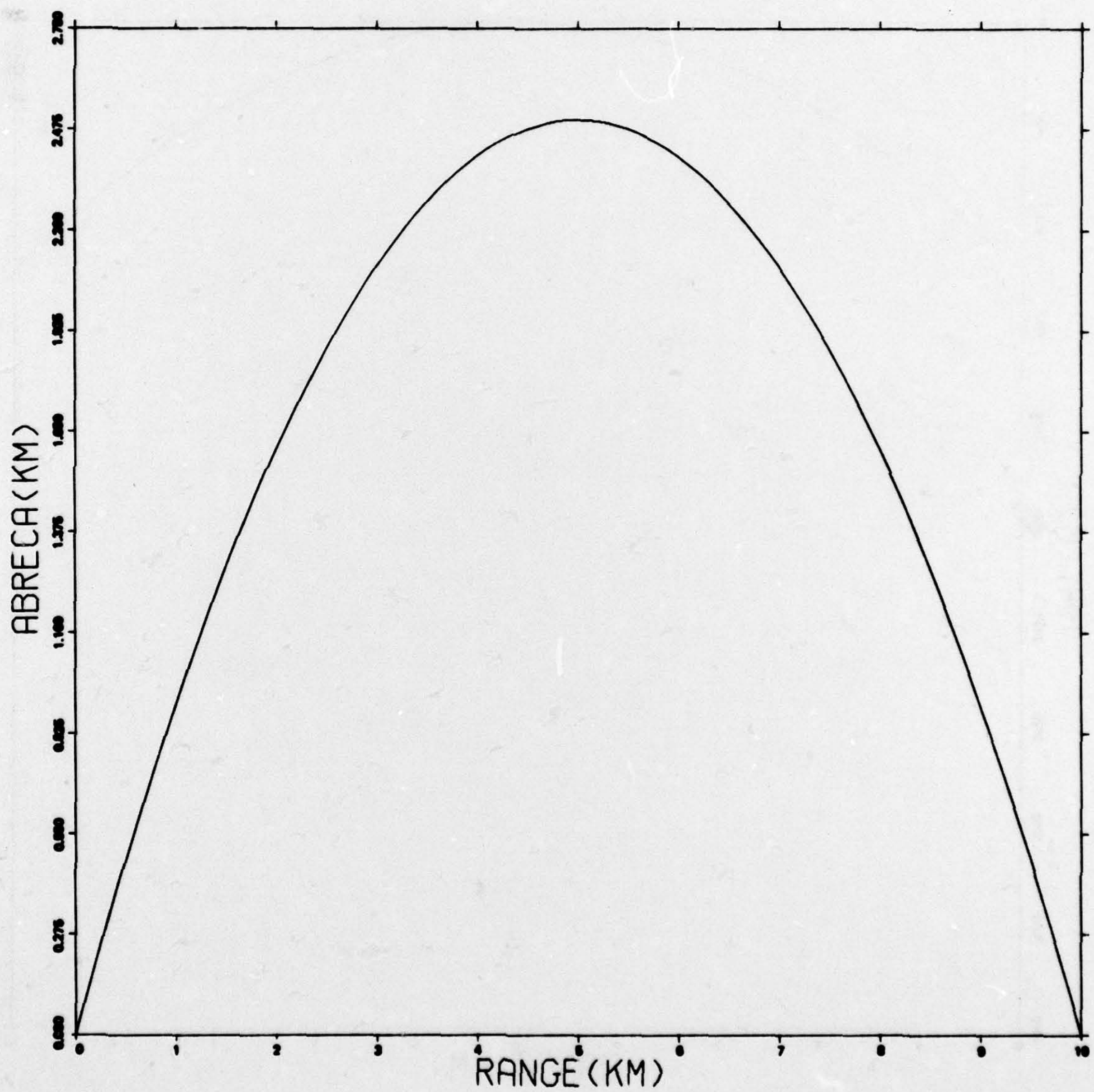

Fig. C4—Plot of the absolute value of the reciprocal of the phase curvature function

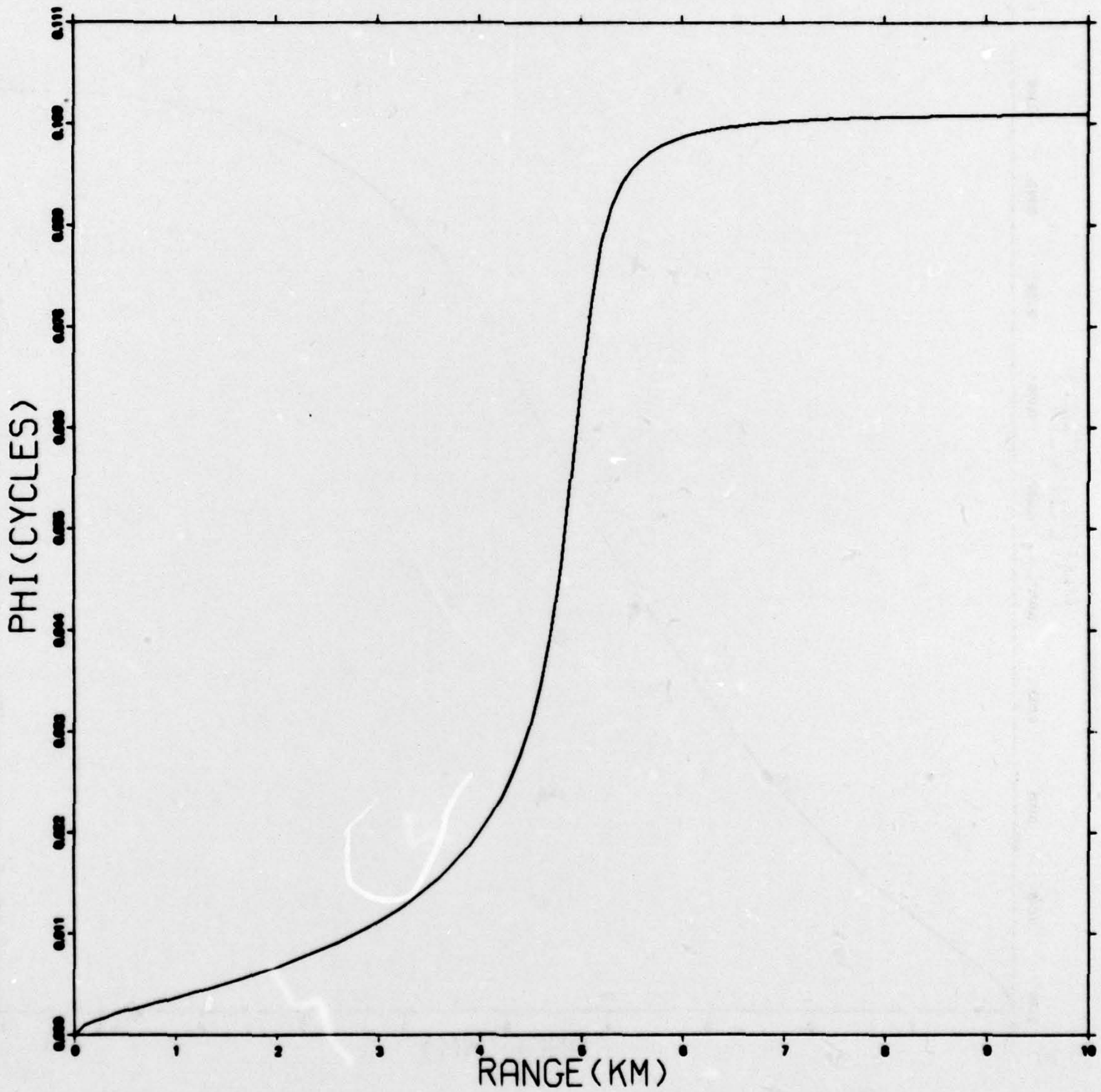

Fig. C5—Plot of the rms phase fluctuation vs horizontal path length

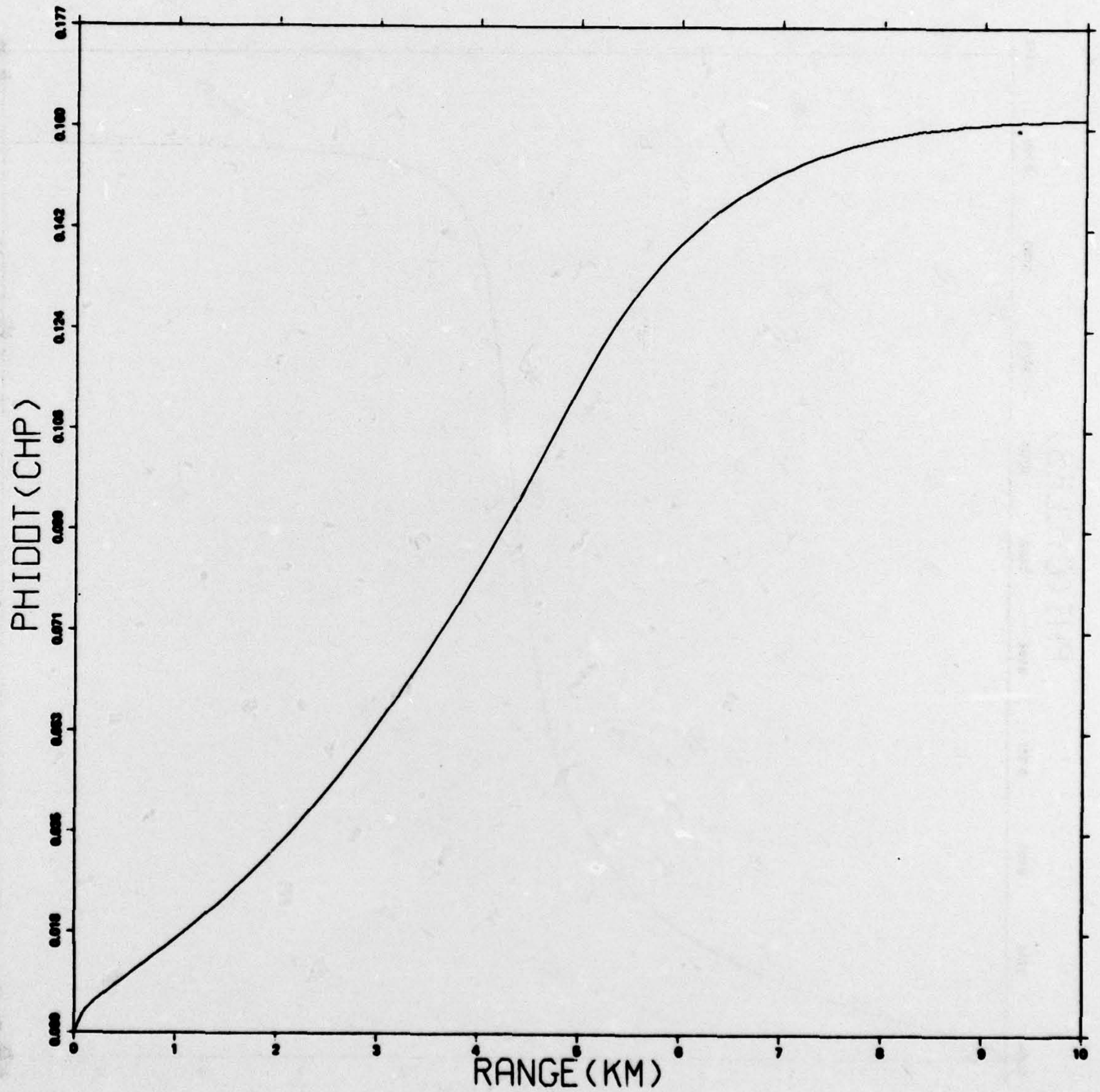

Fig. C6—Plot of the rms phase-rate fluctuation vs horizontal path length

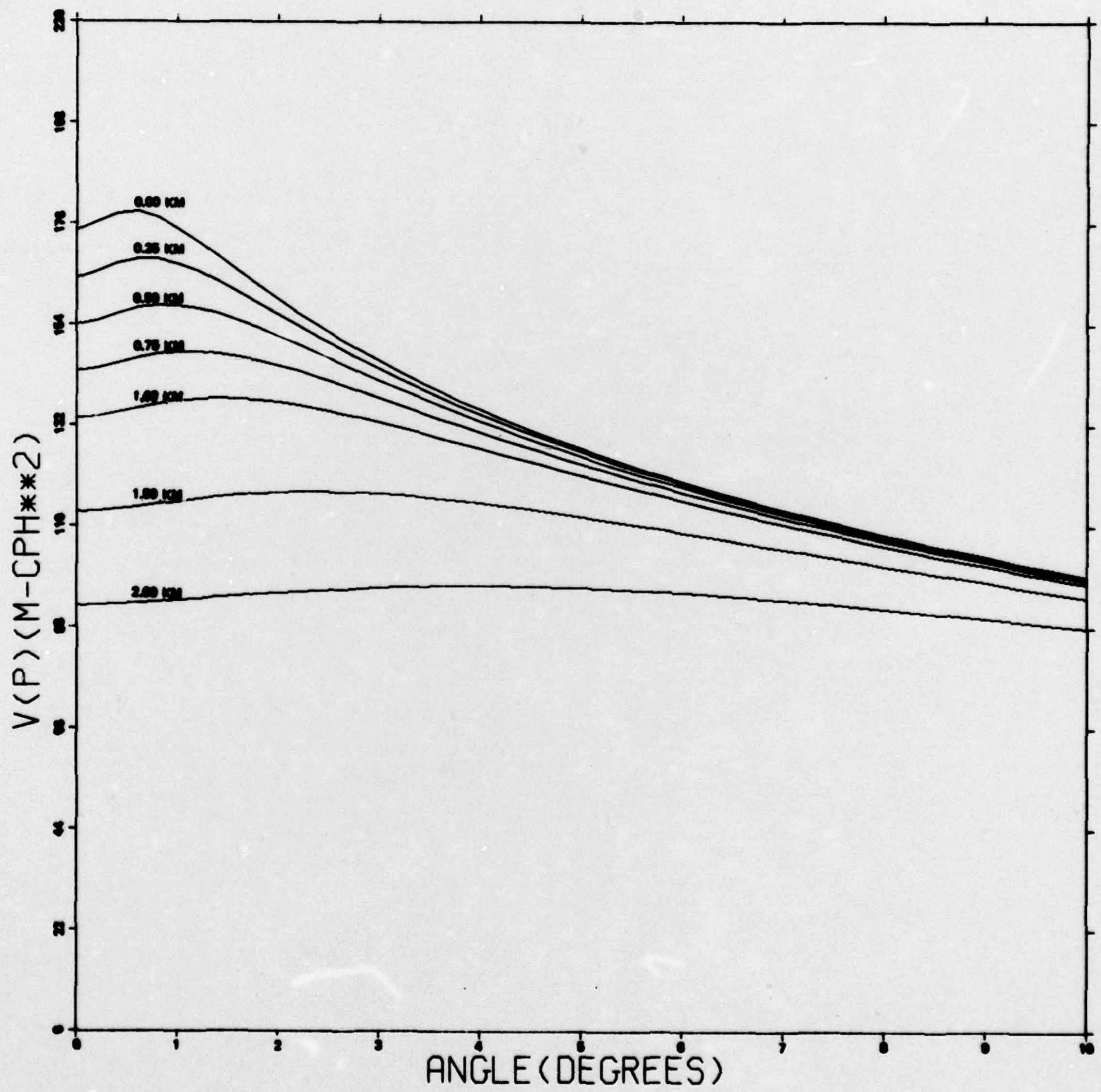

Fig. C7—Plot of $V_p(\theta,z)$ vs θ for various values of z